TECHNICAL CALCULATIONS

BY MEANS OF INTERVAL MATHEMATICS

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Motivation

The idea of this paper is to add some elementary examples of applications to the wide spread and very few others that seem to exist in the interval mathematics literature. The examples presented here are intended to motivate engineer students very early to deal with interval mathematics. They can be inserted into the first lessons of a lecture on calculus, e.g. after having introduced inequalities (and interval arithmetic !), as well as into the exercise collection of a programming course for all sorts of students that need mathematics as an instrument of practical support in their future professions.

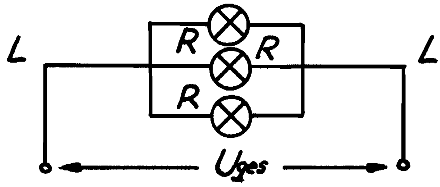
Note that the technical surroundings of the examples are the bridge to the engineer student's ear, whereas the simplicity of the formulas dealt with is the way to surround his fear : Interval mathematics are useful a n d easy !

Interval Arithmetic in HP BASIC

All printed programs or numerical results are produced on the desk top computers HEWLETT PACKARD 9845T of the Mathematisches Labor I, Fachhochschule Darmstadt. The programming language used here is a very high level structured BASIC which is at least as powerful as FORTRAN 77. It is combined with a BASIC-written precompiler of Heinz LERCHE and the author and completed by a BASIC interval arithmetic package developed by the students Marika GAUCH, Bernd GUTHIER, Gerhard HORNBERGS and Reiner UHL and the author. For more information, see /T/.

Electric Current Rules

When studying the elements of electricity, one is faced to many simple formulas which can easily (and should be) written in terms of interval arithmetic. The first example shows how to examplify all f o u r e l e m e n t a r y o p e r a t i o n s at once and how to demonstrate the practical handling of c o n s t a n t s and n-th p o w e r s.



Problem. Three bulbs of R=240Ω resistance each are connected by a L=100m long aluminium line of diameter d=1.5mm and specific resistance ρ=0.02857Ωmm^2/m to a source Uges=220V of electromotive force. The accuracy of all data is $\pm$ 0.5 percent. - Switch on one, two or three lamps. Which is the operating voltage in each case ? (cf. /L/,Nr.947)

<u>Solution</u>. (a) One lamp :

Resistance	:	Rges = Rl + R ,	
current	:	Iges = Uges / Rges ,	1)
voltage drop in the line	:	Ul = Rl * Iges ,	
operating voltage (1 bulb):	:	U1 = Uges - Ul .	

First calculate $\underline{Rl}$ in consideration of all possible errors. Then define

$$\underline{R} := [\ 238.8, 241.2\ ]\ ,\quad \underline{Uges} := [\ 218.9, 221.1\ ]$$

and perform

$$\begin{aligned} \underline{Rges} &= \underline{Rl} \oplus \underline{R}\ , \\ \underline{Iges} &= \underline{Uges} \oslash \underline{Rges}\ , \\ \underline{Ul} &= \underline{Rl} \circledast \underline{Iges}\ , \\ \underline{U1} &= \underline{Uges} \ominus \underline{Ul}\ . \end{aligned}$$

(The ideal operations $\oplus$, ... , $\oslash$ will later be replaced by the corresponding machine operations that provide rounded interval arithmetic.)

The determination of the line resistance Rl is done as follows :

Line length	:	$Ll = 2 * L$,
sectional area of line	:	$A = \pi * (d / 2)^2$,
line resistance	:	$Rl = (\varrho * Ll) / A$
		$= \varrho * L * 8 / (\pi * d^2)$.

Introduce the interval constant $0.0\underset{\bullet}{2}857$ of width zero and execute

$$\underline{Rho} = 0.0\underset{\bullet}{2}857 \circledast [\ 0.995, 1.005\ ]$$

to fix the possible error of ϱ . After the definition of $\underset{\bullet}{8}$ and $\underline{\pi}$, one gets

$$\underline{Rl} = \underline{Rho} \circledast \underline{L} \circledast \underset{\bullet}{8} \oslash (\underline{\pi} \circledast \underline{d} \circledast \underline{d})$$

with $\underline{d} := 1\underset{\bullet}{.}5 \circledast [\ 0.995, 1.005\ ]$.

(b) Two lamps : The equation

$$Uges = Ul + U12 = Rl * Iges + R * (Iges / 2) = (Rl + R / 2) * Iges$$

leads to

$$\underline{Rges} = \underline{Rl} \oplus \underline{R} \oslash \underset{\bullet}{2}$$

which replaces the corresponding formula in (a).

(c) Three lamps : Since

$$Uges = Ul + U123 = Rl * Iges + R * (Iges / 3) = (Rl + R / 3) * Iges\ ,$$

now

$$\underline{Rges} = \underline{Rl} \oplus \underline{R} \oslash \underset{\bullet}{3}$$

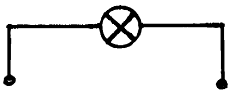
holds.

(d) One to three lamps : All cases can be treated s i m u l t a n e o u s l y when using interval arithmetic :

$$\underline{Rges} = \underline{Rl} \oplus \underline{R} \circledast [\ 1/3\ ,\ 1\ ]\ .$$

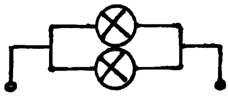
<u>Numerical results.</u>

R	[2.38799999999E+02	, 2.41200000001E+02]
L	[9.94999999999E+01	, 1.00500000001E+02]
D	[1.49249999999E+00	, 1.50750000001E+00]
Rho	[2.84271499999E-02	, 2.87128500001E-02]
Uges	[2.18899999999E+02	, 2.21100000001E+02]
Rl	[3.16943518253E+00	, 3.29878338595E+00]
Rges	[2.41969435181E+02	, 2.44498783388E+02]
Iges	[8.95300978457E-01	, 9.13751771317E-01]
Ul	[2.83759842007E+00	, 3.01426916211E+00]
U1	[2.15885730836E+02	, 2.18262401582E+02]



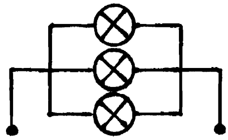
1) The index 'ges' ('gesamt') should be read as 'total'.

Rges	[1.22569435180E+02 ,	1.23898783389E+02]
Iges	[1.76676472529E+00 ,	1.80387549047E+00]
U1	[5.59964627958E+00 ,	5.95059449829E+00]
U 2	[2.12949405500E+02 ,	2.15500353722E+02]



(b)

Rges	[8.27694351820E+01 ,	8.36987833865E+01]
Iges	[2.61533072694E+00 ,	2.67127593073E+00]
U1	[8.28912121991E+00 ,	8.81196065959E+00]
U 3	[2.10088039338E+02 ,	2.12810878782E+02]



(c)

Rges	[8.27694351820E+01 ,	1.23898783389E+02]
Iges	[1.76676472529E+00 ,	2.67127593073E+00]
U1	[5.59964627958E+00 ,	8.81196065959E+00]
U 23	[2.10088039338E+02 ,	2.15500353722E+02]

(b) or (c)

The operating voltage varies between 210 and 218.3 V .

Alternating Current Measuring Bridge

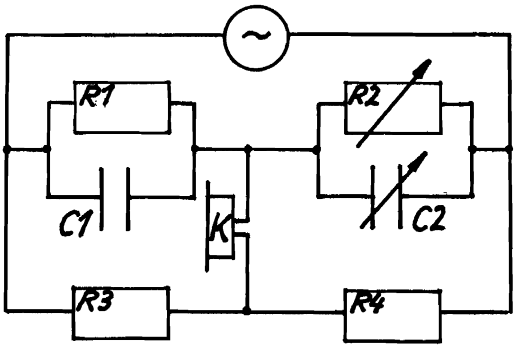
The capacity of the unknown condenser C1 and the resistance of the unknown resistor R1 may be found out by using a circuit as shown. The idea is to balance the variable capacity C2 and the variable resistance R2 until the tone in the earphone K (which must be of little resistance) reaches a minimum or vanishes. In this case,

$$C1 = R4 * C2 / R3$$

and

$$R1 = R3 * R2 / R4$$

hold (cf. /G/, S20).



Problem. Given are two resistors of resistance

$$R3 \in [\, 9.9,10.1 \,]\Omega \quad \text{and } R4 \in [\, 6.8,6.9 \,]\Omega \ ,$$

according to the producers declaration. Due to u n c e r t a i n t i e s o f p e r c e p t i o n , C2 and R2 are estimated by

$$C2 \in [\, 40.2,41.5 \,]F \text{ and } \quad R2 \in [\, 18.3,19.8 \,]\Omega \ .$$

Compute the values of C1 and R1 !

Solution.

```
10   Kanal=7                       ! DRUCKER
20   Adresse=2
30
40   !
50   ! INT INTERVAL R1;R2;R3;R4;C1;C2
60   !
70   ! INT R2:=[18.3,19.8] ! OHM
80   ! INT R3:=[9.9,10.1]  ! OHM
90   ! INT R4:=[6.8,6.9]   ! OHM
100  ! INT C2:=[40.2,41.5] ! FARAD
110  !
120  PRINT "EINGABEDATEN :"
130  PRINT
140  Vorschub=0
150  Genauigkeit=4              ! 4-STELLIGE AUSGABE
160                             ! MIT AUSSENRUNDUNG
170  ! INT OUTPUT R2;R3;R4;C2
180  !
190  ! INT C1=R4/R3*C2
200  ! INT R1=R3/R4*R2
210  !
220  PRINT LIN(2);"AUSGABEDATEN :"
230  PRINT
240  Vorschub=1                 ! JEDE AUSGABE EINZELN
250  Genauigkeit=12             ! VOLLE GENAUIGKEIT
260  ! INT OUTPUT C1;R1
270  END
```

Numerical results.

EINGABEDATEN :

R2 [1.829E+01,1.981E+01]R3 [9.899E+00,1.011E+01]R4 [6.799E+00,6.901E+00]C2 [4.019E+01,4.151E+01]

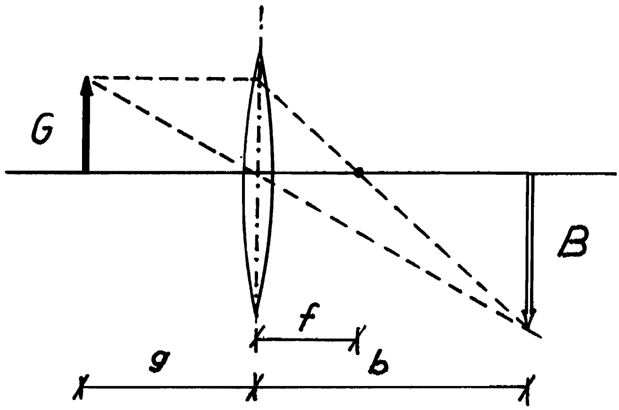
AUSGABEDATEN :

C1	[2.70653465345E+01 ,	2.89242424244E+01]
R1	[2.62565217389E+01 ,	2.94088235298E+01]

The condensers capacity is C1 = (28.0$\pm$1.0)F, the unknown resistance is R1 = (27.9$\pm$1.7)Ω.

Lens Equation

This example may convince all engineers who "believe" in the classical error estimation method.



G : object

B : image

f : focal length

g : object distance

b : image distance

Lens eqation for thin lenses :

1 / f = 1 / b + 1 / g .

Problem. Let f = (20$\pm$1)cm be the focal length of a thin lens. The image distance b has been metered to b = (25$\pm$1)cm. - How large is the distance between the object and the lens ?

Solution. This question is usually handled as follows :

$$g = g(f,b) = 1 / (1 / f - 1 / b),$$
$$g_o = g(f_o,b_o),$$
$$g \doteq g_o \pm \Delta g$$

with $\Delta g = (1 - f_o / b_o)^{-2} * \Delta f + (b_o / f_o - 1)^{-2} * \Delta b$.

In this case, one has f_o = 20cm, b_o = 25cm, $\Delta f = \Delta b = 1$. Hence, $g_o = 1 / (1/20 - 1/25) = 100$, $\Delta g = (1-20/25)^{-2} + (25/20-1)^{-2} = 41$, $g \doteq (100 \pm 41)$cm or

$$g \in [59,141]\text{cm} .$$

This result is w r o n g !

Calculate instead

$$g \in \lfloor g \rfloor = \underset{\bullet}{1} \oslash (\underset{\bullet}{1} \oslash \lfloor f \rfloor \ominus \underset{\bullet}{1} \oslash \lfloor b \rfloor)$$

which leads to the (c o r r e c t) statement

$$g \in [\, 70.5,168.1 \,] \text{ cm} .$$

It can easily be seen that the endpoints of this interval (see the more precise representation in the computer output beneath or the abbreviated version above) are nearly sharp, i.e. the values can be taken leaving rounding effects aside.

The latter is not true for the result produced with the interval version of the algebraically slightly transformed equation

$$g = (b * f) / (b - f).$$

This observation helps to explain the need of dependent interval arithmetic or, for practical reasons, of special a l g e b r a i c t r a n s f o r m a t i o n s before evaluating formulas (see the following examples).

Numerical results.

```
60    !
70    ! INT INTERVAL Bildweite_b;Brennweite_f;Ggnstandswte_g;Eins
80    !
90    ! INT Bildweite_b:=[24,26]
100   ! INT Brennweite_f:=[19,21]
110   ! INT Eins:=[1]
120   !
130   PRINT "EINGABEDATEN :"
140   PRINT
150   ! INT OUTPUT Bildweite_b;Brennweite_f
160   !
170   ! INT Ggnstandswte_g=Eins/(Eins/Brennweite_f-Eins/Bildweite_b)
180   ! INT Grosses_g=Bildweite_b*Brennweite_f/(Bildweite_b-Brennweite_f)
190   !
200   PRINT LIN(2);"AUSGABEDATEN :"
210   PRINT
220   ! INT OUTPUT Ggnstandswte_g;Grosses_g
230   END
```

```
EINGABEDATEN :

Bildweite_b          [ 2.40000000000E+01 , 2.60000000000E+01 ]
Brennweite_f         [ 1.90000000000E+01 , 2.10000000000E+01 ]


AUSGABEDATEN :

Ggnstandswte_g       [ 7.05714285695E+01 , 1.68000000009E+02 ]
Grosses_g            [ 6.51428571425E+01 , 1.82000000002E+02 ]
```

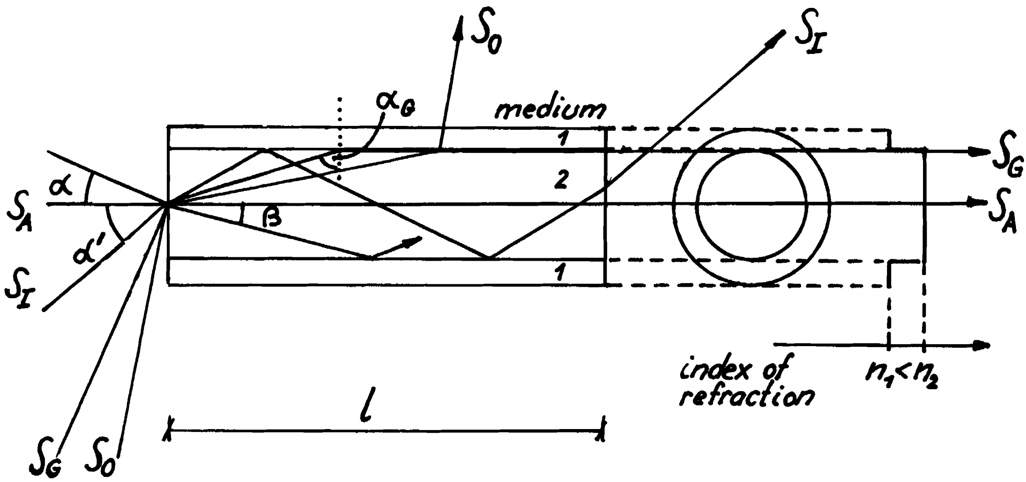
The idea of this problem has been given to the author by his student Berthold SCHOLL.

Usable Frequency Range of Fiber Optical Waveguides

Look at an optical fiber line of length l with a steplike profile of indices of refraction (next page). According to SNELLIUS, one has

$$\sin\alpha / \sin\beta = c_o / c_2 = n_2 / n_o = n_2$$

with c_o velocity of light and n_o aerial index of refraction. The velocity of an axial light pulse is given by $v_2 = c_2 = c_o / n_2$, its time of



S_A : axial ray, S_G : marginal ray of total reflection, α_G : marginal angle of total reflection, $n_{\bullet}$ or $c_{\bullet}$: index of refraction or velocity of light of medium 0 (air) or 1 or 2 (fiber), respectively.

---

transit is given by $t_A = l \,/\, v_2$. The time of transit of a light pulse on the marginal ray S_G is determined by

$$t_G = l \,/\, (v_2 * \sin\alpha_G) = (l * n_2) \,/\, (v_2 * n_1) \,.$$

Every pulse transmitted at the input side will be received distorted (broadened) on the output side of the line. This is due to the running time difference

$$t_G - t_A > 0.$$

It means that the line cannot be used to transmit pulse sequences of arbitrary choosen frequencies. The usable band width b is to be calculated as follows :

$$b = \frac{1}{2*(t_G - t_A)} = \frac{1}{2*\left(\dfrac{l*n_2}{v_2*n_1} - \dfrac{l}{v_2} \right)} = \frac{v_2}{2*l*\left(\dfrac{n_2}{n_1} - 1 \right)} \,,$$

$$b = c_o \,/\, (2 * l * n_2 * (n_2 \,/\, n_1 - 1))$$

(cf. /C/).

<u>Problem</u>. Given a fiber optical waveguide of lenght $l = (220 \pm 0.2)$m. The values of its indices of refraction (steplike index profile assumed) are known with an accuracy of 1 per mil : $n_1 \doteq 1.51$, $n_2 \doteq 1.58$. Estimate the usable band width b by computing

$\lfloor b1 \rfloor = \lfloor co \rfloor \oslash (2 \circledast \lfloor l \rfloor \oslash \lfloor n1 \rfloor \circledast \lfloor n2 \rfloor^2 - 2 \circledast \lfloor l \rfloor \circledast \lfloor n2 \rfloor)$,

$\lfloor b2 \rfloor = \lfloor co \rfloor \oslash (2 \circledast \lfloor l \rfloor \circledast (\lfloor n2 \rfloor^2 \oslash \lfloor n1 \rfloor - \lfloor n2 \rfloor))$ and

$\lfloor b3 \rfloor = \lfloor co \rfloor \oslash (2 \circledast \lfloor l \rfloor \circledast (\lfloor n2 \rfloor \oslash \lfloor n1 \rfloor - 1) \circledast \lfloor n2 \rfloor)$.

According to newer mensurations, the velocity c_o of light comes to $(299\ 792\ 456.2 \pm 1.1)$m/s .

Numerical results.

L	[2.19800000000E+02 ,	2.20200000000E+02]
N1	[1.50848999999E+00 ,	1.51151000001E+00]
N2	[1.5784;999999E+00 ,	1.58158000001E+00]
C0	[2.99792455100E+08 ,	2.99792457300E+08]
B1	[8.23540866264E+06 ,	1.06834884099E+07]
B2	[8.53134233801E+06 ,	1.02225602797E+07]
B3	[8.88314390012E+06 ,	9.76023867943E+06]
B	[8.88314390012E+06 ,	9.76023867943E+06]

There is a guaranteed band width of 8.8MHz. This value is sufficient to perform a telephone communication (3.1kHz) or to transmit music (15 kHz) or a TV program (6 MHz).

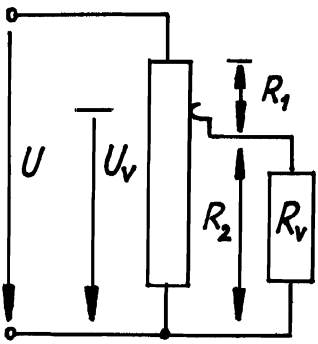
Note that the result with the largest l e f t endpoint (which needs not necessarily to be the result of smallest interval width to fit the practical aspect of the question) is the most relevant one. The interval

$$[\ 0\ ,\ \lfloor b3 \rfloor\ ]$$

represents all possible frequencies the fiber optic waveguide may be used for.

Bleeding an Electrical Potential

The configuration shown may be used to transform a given voltage U to a consumer voltage Uv (at the load resistor Rv). It is given by



$$U_v = \frac{R_2 * R_v}{R_1 * R_2 + R_1 * R_v + R_2 * R_v} * U\ ,$$

see /G/, chapter S8.

Problem. What is the range of the consumer potential Uv in a voltage-divider circuit as shown in the figure if U $\in$ [210,230]V, Rv $\in$ [900,910]Ω , R1 $\in$ [9.9,10.1]Ω and R2 $\in$ [19.8,20.2]Ω ?

Solution. Reduce the formula to the algebraically equivalent form

$$U_v = U\ /\ (\ R_1 * (\ 1\ /\ R_v + 1\ /\ R_2\ ) + 1\ ).$$

Since each variable does not occur but once in the expression, its interval version will not produce an overestimation.

```
50    ! INT INTERVAL R1;R2;Rv;U;Uv;Eins
60    !
70    ! INT R1:=[9.9,10.1]   ! OHM
80    ! INT R2:=[19.8,20.2]  ! OHM
90    ! INT Rv:=[900,910]    ! OHM
100   ! INT U:=[210,230]     ! VOLT
110   ! INT Eins:=[1]
120   !
130   Genauigkeit=4          ! 4-STELLIGE AUSGABE
140                          ! MIT AUSSENRUNDUNG
150   ! INT OUTPUT R1;R2;Rv;U
```

```
160   !
170   ! INT Uv=R2*Rv*U/(R1*R2+R1*Rv+R2*Rv)
180   PRINT LIN(2);"Rv naiv"
190   Vorschub=1             ! JEDE AUSGABE EINZELN
200   Genauigkeit=12         ! VOLLE GENAUIGKEIT
210   ! INT OUTPUT Uv
220   !
230   ! INT Uv=U/(R1*(Eins/Rv+Eins/R2)+Eins)
240   PRINT LIN(2);"Rv optimiert"
250   ! INT OUTPUT Uv
260   !
270   END
```

Numerical results.

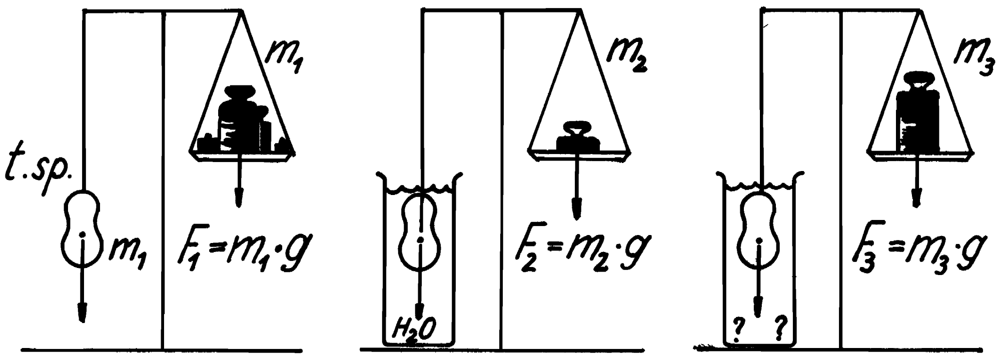
```
R1                  [9.899E+00,1.011E+01]
R2                  [1.979E+01,2.021E+01]
Rv                  [8.999E+02,9.101E+02]
U                   [2.099E+02,2.301E+02]


Rv naiv
Uv                  [ 1.34722875237E+02 , 1.57017635733E+02 ]


Rv optimiert
Uv                  [ 1.38037726326E+02 , 1.53233411791E+02 ]
```

The consumer voltage will vary between 138.0V and 153.3V .

Density Determination of an Unknown Fluid

This example demonstrates the influence of "implicit" constants that can but have not been removed before evaluating a formula. Furthermore, it can be used to teach that sometimes there are error dependent optimal algebraic transformations in absence of a total reduction as could be used in the examples above.

Take a test specimen and handle it as follows :



1. Weighing in the air. 2. Weighing in distilled water 3. Weighing in the unknown fluid

Any body of volume V in a fluid of density ρ meets a lifting force

$$- \rho * V * g ,$$

where g is the acceleration of the fall. Formally, the following holds :

$$\rho = \rho * 1 = \rho * \frac{V * g}{V * g} = \frac{- \rho * V * g}{- 1 * V * g} .$$

Being aware of the fact that the density of water is $\rho_{H2O} = 1$, this leads to

$$\rho_? = \frac{- \rho_? * V * g}{- \rho_{H2O} * V * g} = \frac{F_3 - F_1}{F_2 - F_1} .$$

Problem. Determine the density of an unknown fluid by an amber cube of edge length 1cm using the method explained above. The cube's weight m1 is about 1. Since the density of amber is nearly the same as that of water, $0 < m2 \doteq 0$ will be observed. Let the cube's weight be $m3 \doteq 0.3$ if it is circumcirculated by the unknown fluid. - All masses are given in grams. Assume at first an accuracy of $\pm$ 0.001, then change the error successively for each mass from $\pm$ 0.001 to $\pm$ 0.005. Compute for all four cases four different values :

$$\llcorner \rho 1 \lrcorner = (\llcorner F3 \lrcorner \ominus \llcorner F1 \lrcorner) \oslash (\llcorner F2 \lrcorner \ominus \llcorner F1 \lrcorner) ,$$
$$\llcorner \rho 2 \lrcorner = (\llcorner m1 \lrcorner \ominus \llcorner m3 \lrcorner) \oslash (\llcorner m1 \lrcorner \ominus \llcorner m2 \lrcorner) ,$$
$$\llcorner \rho 3 \lrcorner = \underset{\bullet}{1} \oplus (\llcorner m2 \lrcorner \ominus \llcorner m3 \lrcorner) \oslash (\llcorner m1 \lrcorner \ominus \llcorner m2 \lrcorner) ,$$
$$\llcorner \rho 4 \lrcorner = \underset{\bullet}{1} \oslash (\underset{\bullet}{1} \oplus (\llcorner m3 \lrcorner \ominus \llcorner m2 \lrcorner) \oslash (\llcorner m1 \lrcorner - \llcorner m3 \lrcorner)) .$$

Solution.

```
30   !
40   ! INT INTERVAL F<1:3,1:4>;M<1:3,1:4>;Rho<1:4,1:4>
50   ! INT INTERVAL Mwert<3>;Fehler;Eins;G
60   !
70   ! INT Mwert<1>:=[.999,1.001]        ! MASSEN GENAU
80   ! INT Mwert<2>:=[0,.001]
90   ! INT Mwert<3>:=[.299,.301]
100  ! INT Fehler:=[-.004,.004]          ! FEHLERZUSCHALG
110  ! INT G:=[9.8063,9.8151]            ! FALLBESCHLEUNIGUNG
120  ! INT Eins:=[1]                     ! PUNKTINTERVALL
130  !
140  FOR I=1 TO 4
150     FOR J=1 TO 3
160        ! INT M<J,I>:=Mwert<J>        ! MASSEN DEFINIEREN
170     NEXT J
180     IF I>1 THEN
190        K=I-1
200        ! INT M<K,I>=M<K,I>+Fehler    ! FEHLER ANBRINGEN
210        IF K=2 THEN M(2,I,1)=0        ! MASSE NICHT NEGATIV
220     END IF
230  NEXT I
240  FOR I=1 TO 4
250     FOR J=1 TO 3
260        ! INT F<J,I>=M<J,I>*G         ! KRAEFTE BERECHNEN
270     NEXT J
280  NEXT I
290  FOR I=1 TO 4                        ! SPEZ. GEWICHT
300     ! INT  Rho<1,I>=(F<3,I>-F<1,I>)/(F<2,I>-F<1,I>)
310     ! INT  Rho<2,I>=(M<1,I>-M<3,I>)/(M<1,I>-M<2,I>)
320     ! INT  Rho<3,I>=Eins+(M<2,I>-M<3,I>)/(M<1,I>-M<2,I>)
330     ! INT  Rho<4,I>=Eins/(Eins+(M<3,I>-M<2,I>)/(M<1,I>-M<3,I>))
340  NEXT I
350  !
```

Numerical results.

M1	[9.989E-01, 1.002E+00]
M2	[-1.000E-99, 1.001E-03]
M3	[2.989E-01, 3.011E-01]
Rho1	[6.963E-01, 7.044E-01]
Rho2	[6.972E-01, 7.035E-01]
Rho3	[6.983E-01, 7.024E-01]
Rho4	[6.986E-01, 7.021E-01]

——— worst of all results
······· worst result leaving Rho1 aside

M1	[9.949E-01, 1.006E+00]	[9.989E-01, 1.002E+00]	[9.989E-01, 1.002E+00]
M2	[-1.000E-99, 1.001E-03]	[-1.000E-99, 5.001E-03]	[-1.000E-99, 1.001E-03]
M3	[2.989E-01, 3.011E-01]	[2.989E-01, 3.011E-01]	[2.949E-01, 3.051E-01]
Rho1	[6.896E-01, 7.113E-01]	[6.963E-01, 7.072E-01]	[6.923E-01, 7.084E-01]
Rho2	[6.904E-01, 7.104E-01]	[6.972E-01, 7.063E-01]	[6.932E-01, 7.075E-01]
Rho3	[6.971E-01, 7.036E-01]	[6.971E-01, 7.064E-01]	[6.943E-01, 7.064E-01]
Rho4	[6.974E-01, 7.033E-01]	[6.986E-01, 7.049E-01]	[6.946E-01, 7.061E-01]

It is obvious, that the formula for $\lfloor \varrho 1 \rfloor$ is bad in all cases since the superfluous constant g has not been removed in time . More surprising is, that $\lfloor \varrho 4 \rfloor$ produces (although using 5 instead of 4 or 3 arithmetic operations !) the best result for all cases. It does so even if the error of m3 is the largest one. This is a contradiction to the idea that the number of occurrences of a large width interval variable in a formula should be minimized in any case.

Remark : For the choice of the interval $\lfloor g \rfloor$ (acceleration of the fall) see the last example. - Known fluid densities next to the results above are those of petro ether (0.67kg/dm^3), benzine or hydrocyanic acid (0.7 kg/dm^3 each), ether (0.73kg/dm^3) and alcohol or acetone (0.79kg/dm^3 each). If the fluid is no emulsion and the experimenter is still alive, all results indicate benzine.

Hydrostatic Pressure in Open Reservoirs

The last example may help to illustrate why there is a need for functions of type $\mathbb{R} \rightarrow I(\mathbb{R})$ or even $I(\mathbb{R}) \rightarrow I(\mathbb{R})$.

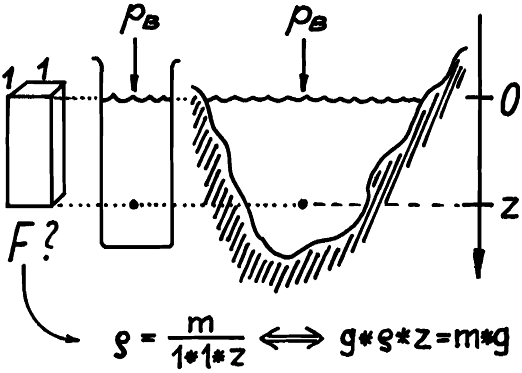
Consider a fluid of known density ϱ in an open reservoir. Let the bathometer be normed as shown in the figure. Let p_B be the hydrostatic pressure at z = 0. Then follows for the hydrostatic pressure

$$p(z) = p_B + g * \varrho * z .$$

If $p_B \in \lfloor ao \rfloor$ and $g * \varrho \in \lfloor a1 \rfloor$, one has

$$p(z) \in \lfloor a0 \rfloor \oplus \lfloor a1 \rfloor \circledast \underset{\bullet}{z}$$

with an s t r a i g h t l i n e i n t e r v a l on the right hand side.



Problem. Plot a diagram / calculate a table for a diver to show the underwater pressure for bar ometric air pressures between 930mbar and 1070 mbar (HPa) and depths of water between 0m and 100m. - Take into account, that the density of natural water (depending upon its salt content) varies between 0.99kg/dm³ and 1.03kg/dm³ and that the acceleration g of the fall differs in its value depending on the terrestrial latitude :

Latitude	0°	20°	D 40°	D 45°	D 50°	55°	60°	90°
m/s^2 $\pm$1E-4	9.7805	9.7865	9.8018	9.8063	9.8108	9.8151	9.8192	9.8322

D : Germany

(cf. /GE/). - For practical reasons, use steps of 20 for the barometric air pressure.

Solution.

```
10     ! INT INTERVAL P;Pb;G;Rho;Z;Anstieg;Zuschlag   ! DEKLARATION
20     ! INT G:=[9.7804,9.8323]                       ! FALLBESCHLEUNIGUNG
30     ! INT Rho:=[.99,1.03]                          ! DICHTE H20
40     ! INT Anstieg=G*Rho                            ! GERADENANSTIEG
50     !
        . . .
420     !
430     ! INT Zuschlag:=Anstieg                       ! GERADEN ZEICHNEN
440     Zuschlag(1)=Zuschlag(1)*100
450     Zuschlag(2)=Zuschlag(2)*100
460     FOR X=930 TO 1050 STEP 20
470        ! INT Pb:=[X,X+20]
480        ! INT P=Pb+Zuschlag
490        MOVE Pb(1),0                               ! LINKE ECKE
500        LINE TYPE 1
510        DRAW P(1),-100
520        MOVE P(2),-100                             ! RECHTE ECKE
530        LINE TYPE 4
540        DRAW Pb(2),0
550     NEXT X
560     DUMP GRAPHICS #Kanal,Adresse
570     END
```

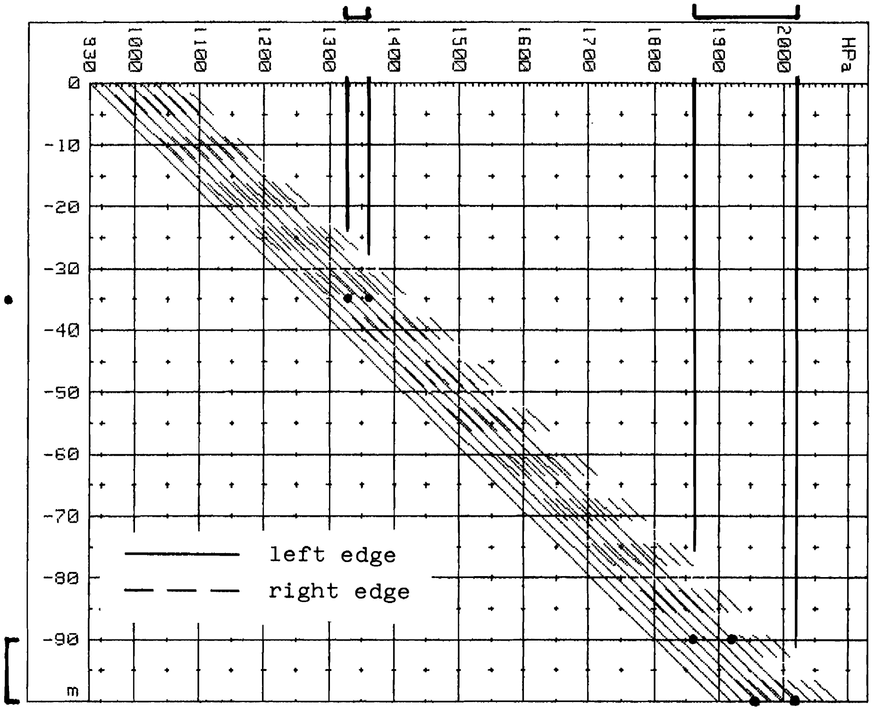
(All parts of the program that do not use interval arithmetic have been suppressed.)

Graphic result. The diagram is given on the next page.

A pearl-fisher will not reach a deepness of more than 35m. Assume a barometric air pressure of 1000mbar. The diagram says that he will suffer a water pressure of at most between 1320mbar and 1370mbar.

A man who uses a diving dress may reach 90m to 100m. According to the diagram, he will find a water pressure of between 1860mbar and 2030mbar at that depth.

It might be interesting to plot one straight line interval for PICARD and WALSH that reached in 1960 with their bathyscaph "Trieste" the depth of 10912m. Since the air pressure of the two days experiment may not be available, take [930,1070]mbar, the interval of all possible natural values. Otherwise take the interval observed during the 22nd and 23rd of January 1960 by the experimenters.



For more examples see /T/.

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Remark : All the standard literature on interval mathematics needed in this paper has been ommitted for the sake of shortness.