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Model reduction of uncertain systems retaining the uncertainty structure

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Abstract

Model reduction of high order linear-in-parameters discrete-time systems is considered. The main novelty of the paper is that the coefficients of the original system model are assumed to be known only within given intervals, and the coefficients of the derived reduced order model are also obtained in intervals, such that the complex value sets of the uncertain original and reduced models will be optimally close to each other on the unit circle. The issue of inclusion of one value set in another is also addressed in the paper. The meaning of model reduction is defined for linear-in-parameters systems. The algorithm for obtaining the value sets of such systems is derived in the paper. Then, applying a novel approach, the infinity norm of "distance" between two polygons representing the original and the reduced uncertain systems is minimized. A noteworthy point is that by a special definition of this distance the problem is formulated as a linear semi-infinite programming problem with linear constraints, thus reducing significantly the computational complexity. Numerical example is provided.

Keywords: Uncertain systems; Model reduction; Linear semi-infinite programming; Linear-in-parameters interval systems

1. Introduction

Many different methods of model reduction are heavily exploited by control engineers in the framework of classical control when the original model is assumed absolutely known and fixed [6,9,12,15–17,21–23,25]. However, in most "real life" engineering systems, there are many sources of uncertainties and inaccuracies about the exact values of the system's parameters.

Then, the basic question is: what is the meaning of model reduction for systems with uncertain parameters?

This has not yet been thoroughly defined in the literature. Do we want the reduced order model to be a *fixed-coefficients* model approximating some kind of a "nominal" model, or do we want the reduced order model to be *uncertain* as well? Both possibilities have their own applications and they are both equally important. The fixed-coefficients reduced model is applicable when it is needed to simplify the complexities of

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the original system for purposes of simulation or analysis or design. It is absolutely necessary if it relates to a controller which actually has to be realized. On the other hand, the uncertain reduced model is applicable when it is needed for stability analysis or for a worst case analysis or for any other purpose of simulation and design where retaining the uncertainty structure is necessary.

Previous publications which relate to the problem of model reduction of systems with uncertainties are very few [2–4,7,10,14,18,20,24]. In [4], the balanced truncation method is extended for uncertain systems using linear matrix inequalities (LMIs) with constraints on the minimal eigenvalue. In [14] the issue of inclusion of the original set in the reduced one is addressed using an iterative algorithm. In [2,24] the iterative methods based on LMIs are presented resulting in local convergence. All the above algorithms are characterized by high computational complexity. In [20,10] the set-membership approach is utilized to obtain the reduced order model consistent with the measurements and the assumptions on the noise. However, [20,10] solve only a problem of inclusion of one specific value set in another, and not their maximal similarity. Additionally, [10] provides only a suboptimal solution to this problem and [20] finds just the value sets consistent with the reduced system model and not the coefficients of the reduced system. In [18] the "dominant eigenvalue" method is extended using interval arithmetics. In [3], the Routh–Pade approximation method is extended for uncertain systems using interval arithmetics. Naturally, the use of interval arithmetics results in conservative solutions. In [7], the first meaning of model reduction of uncertain systems is treated. A method to reduce an interval FIR system to a fixed-coefficients system has been derived, applying a useful novel definition of the "distance" between the original and the reduced models. In the present paper, the second meaning of model reduction of uncertain systems is treated. The coefficients of the derived reduced order model are obtained in intervals, such that the complex value sets of the uncertain original and reduced model will be optimally close to each other, on the unit circle.

The derived method yields optimal results, with regard to the specified criterion, for a class of single-input single-output, linear, shift-invariant, discrete-time systems. The considered class of transfer functions of the system, to be defined in the next section, includes, beside the interval polynomials (FIR interval systems), polynomials whose coefficients are linear functions of interval parameters, rational functions with fixed denominator, and more. Furthermore, an arbitrary interval rational transfer function can also be treated by our method, but in a sub-optimal way: firstly reduce the order of the denominator polynomial by a fixed coefficients polynomial using the method in [7], then use the method derived in the present paper to reduce the rational function with fixed denominator.

Our method is based on an appropriate novel definition of the "distance" between the two uncertain systems: the original one and the reduced one. The definition is such that when the value sets of the two systems are identical at a certain frequency, the distance becomes zero at this frequency. This is not the case in [4], for example. Also, if the uncertainty of the systems tends to zero, the definition of the distance becomes that of the geometrical distance between two points in the complex plane, as is the case in model reduction of fixed-coefficients systems.

The formulation of the method is carried out in such a way that it becomes a *linear* semi-infinite programming problem. This reduces significantly the computational complexity.

To summarize the above, the main contributions of the present paper are:

1. An uncertain system model is reduced, retaining the approximate original complex-valued uncertainty, in the frequency domain. In [7], an uncertain system model was reduced to a fixed-coefficients model, without uncertainty. These two different objectives are almost opposite.
2. The derived method is applicable to any transfer function whose uncertainty is linear-in-parameters and, in a sub-optimal way, to any rational interval function. In [7], only interval polynomials were considered.
3. Although in the present case the value-set of the reduced model at each frequency is a convex polygon in the complex plane and not a point as is the case in [7], here, too, the formulation is carried out in such a way that it becomes a linear semi-infinite programming problem. Hence, the computational complexity is significantly reduced in the same manner as done in the simpler case of [7].

The structure of the paper is as follows. In Section 2 we formulate the problem of model reduction. Then, in Section 3 we derive an algorithm for obtaining the value set of uncertain linear-in-parameters system at each frequency. Based on the results of Section 3 we formulate in Section 4 a linear semi-infinite programming problem which is equivalent to the original model reduction problem. Finally, example is provided in Section 5. This is an illustrative example of IIR system, which indicates the effectiveness of the method. We conclude in Section 6.

2. Problem formulation

Consider systems with the following class of transfer functions

$$f(z) = \sum_{i=0}^{N} n_i\, N_i(z), n_i \in [\underline{n_i}, \overline{n_i}], \quad \underline{n_i}, \overline{n_i} \in \mathbb{R}, \quad (1)$$

where $N_i(z)$ are any given continuous and derivable functions of the complex variable z, and $\underline{n_i}$, $\overline{n_i}$, $i = 0, \ldots, N$ are given real numbers, where $\overline{n_i} \geqslant \underline{n_i}$. Let N' denote the number of interval coefficients of $f(z)$, i.e., those for which $\overline{n_i} > \underline{n_i}$.

Several examples of frequently used $N_i(z)$ are: $N_i(z) = z^{-i}$, $N_i(z) = \sum_{j=0}^{C} c_j^{(i)} z^{-j}$ and $N_i(z) = z^{-i}/\sum_{k=0}^{D} d_k z^{-k}$.

We wish to find the reduced system of the form

$$\tilde{f}(z) = \sum_{i=0}^{X} x_i\, X_i(z), \quad x_i \in [\underline{x_i}, \overline{x_i}], \quad \underline{x_i}, \overline{x_i} \in \mathbb{R}, \quad (2)$$

where $X < N$ is a pre-determined number, $X_i(z)$ are any given continuous and derivable functions of the complex variable z and $\underline{x_i}$, $\overline{x_i}$, $i = 0, \ldots, X$ are to be determined.

In what follows we define the measure of similarity between two uncertain systems of the form (1) and (2), thus formulating the purpose of model reduction for such systems.

We will adopt the idea of infinity-norm of error to define the cost function for measuring the quality of approximation of the original system. Mathematically, we want to minimize the

$$\max_{\theta} d(f(\mathrm{e}^{j\theta}), \tilde{f}(\mathrm{e}^{j\theta})), \quad (3)$$

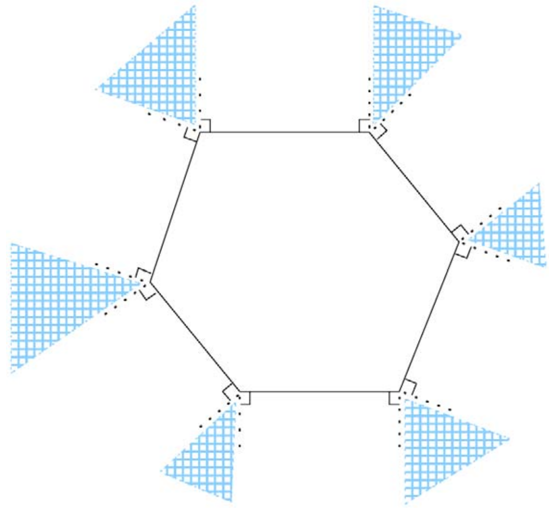

Fig. 1. Example value set of a linear-in-parameters system together with corresponding shaded-sectors containing (9).

where d denotes "distance". It remains to define the distance $d(f(\mathrm{e}^{j\theta}), \tilde{f}(\mathrm{e}^{j\theta}))$ between the original uncertain system and the reduced uncertain system at each frequency θ.

The value sets of both the original (1) and the reduced (2) systems are represented by convex combinations of points in the complex plane. Consider, for example, (2). It is represented by a convex combination of points $\sum_{i=0}^{X} \check{x}_i X_i(z)$ where $\check{x}_i$ is either $\underline{x_i}$ or $\overline{x_i}$. Thus, the value set of any uncertain system of the form (1) or (2) is represented, for each frequency $\theta = \theta_0$ (obviously, $z = \mathrm{e}^{j\theta}$), by a convex polygon in the complex plane (Fig. 1).

Given two polygons representing (1) and (2), let us define the distance between them to be the *maximum* of the following set of distances: (1) the distances from each vertex of polygon 1 to the polygon 2, (2) the distances from each vertex of polygon 2 to the polygon 1. For several examples of the distance between polygons see Fig. 2. The distance so defined is zero for identical polygons and is gradually increasing for larger differences between the polygons. Furthermore, this definition agrees with the usual definition of distance in the degenerate case of zero uncertainty in both the original and the reduced systems. By requiring to minimize this distance we effectively demand the first polygon to be maximally inside the second polygon

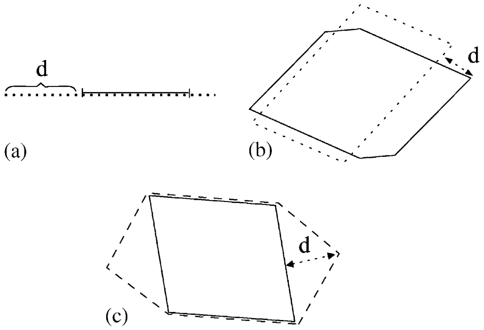

Fig. 2. Examples explaining the definition of distance between polygons.

and at the same time we demand the second polygon to be maximally inside the first one.

Note that the distance between a point and a polygon (defined to be the geometrical distance to the closest point of the polygon) is a nonlinear function of the coordinates of the point and the vertices of the polygon—mainly because of the operation of absolute value involved in the calculation.

The resulting problem of minimizing $\max_\theta d(f(\mathrm{e}^{j\theta}), \tilde{f}(\mathrm{e}^{j\theta}))$ is an optimization problem with infinite number of nonlinear constraints (for each $\theta \in [-\pi, \pi)$ we demand

$$d(f(\mathrm{e}^{j\theta}), \tilde{f}(\mathrm{e}^{j\theta})) < \max_\theta d(f(\mathrm{e}^{j\theta}), \tilde{f}(\mathrm{e}^{j\theta})), \tag{4}$$

which is nonlinear constraint on $\underline{x_i}$ and $\overline{x_i}$). As mentioned in the Introduction, we will formulate the derivation in such a way that it becomes a linear semi-infinite programming problem with linear constraints, reducing significantly the computational complexity.

For some applications it is important, that the value set of the original system will be contained in the value set of the reduced system at all frequencies, or, on the contrary, that the value set of the reduced system will be contained in the value set of the original system. Our algorithm is capable of coping with this kind of problems too, as remarked at the end of Section 4.

3. Obtaining the value set of linear-in-parameters system

Firstly, we discuss how to obtain the polygon representing an uncertain system of the form (1) at any given frequency θ_0. Obviously, the same procedure is applicable for (2) with appropriate substitutions. Let us assume that the numbering of the polygon vertices is done in the counterclockwise direction. It turns out (see (5) below) that we can first calculate the slopes of the edges of the polygon (independently of the coefficients n_i) and then, depending on the actual coefficients n_i, to obtain the whole polygon. This separation into two stages is very important in the following solution.

Due to the fact that every interval coefficient n_i can take any values in $[\underline{n_i}, \overline{n_i}]$ *independently* of other coefficients, it contributes to the polygon two boundary (i.e., exposed) edges with the slopes $\arg(N_i(\mathrm{e}^{j\theta_0}))$ and $\arg(N_i(\mathrm{e}^{j\theta_0})) + \pi$, where $\arg(\Box)$ denotes the phase of $\Box$. Then, to obtain the order of the slopes when following the boundary of the polygon in the counterclockwise direction, find (by simple sort procedure) the sequences m_i and a_i, so that $\{(\arg(N_{m_i}(\mathrm{e}^{j\theta_0})) + a_i\pi) \bmod 2\pi\}$ form an increasing sequence.

The important conclusion is that for *any* coefficients n_i the following slopes

$$\arg(N_{m_i}(\mathrm{e}^{j\theta_0})) + a_i\pi \tag{5}$$

will be observed when following the boundary of the polygon in the counterclockwise direction.

To determine the values of the coefficients in (1) corresponding to each vertex p_k of the polygon, note that every two adjacent vertices differ by only one coefficient, namely,

$$p_{k+1} - p_k = s_k(\overline{n_{m_k}} - \underline{n_{m_k}})N_{m_k}(\mathrm{e}^{j\theta_0}),$$
$$s_k \in \{-1, 1\}. \tag{6}$$

Consequently, when following the boundary of the polygon in the counterclockwise direction and passing the edge with the slope $\arg(N_{m_k}(\mathrm{e}^{j\theta_0}))$, all the coefficients of the new vertex p_{k+1} will be exactly the same as in the previous vertex p_k, except the coefficient n_{m_k}, which will change from $\underline{n_{m_k}}$ to $\overline{n_{m_k}}$. Similarly, when passing the edge with the slope $\arg(N_{m_k}(\mathrm{e}^{j\theta_0})) + \pi$ the n_{m_k} will change from $\overline{n_{m_k}}$ to $\underline{n_{m_k}}$. It is easy to extend this rule to get all the vertices of the polygon—for each vertex we should note the slopes of previous $N + 1$ edges (i.e., the $N + 1$ edges in the clockwise direction starting from the given vertex) and apply the above rule to calculate the corresponding $N + 1$ coefficients

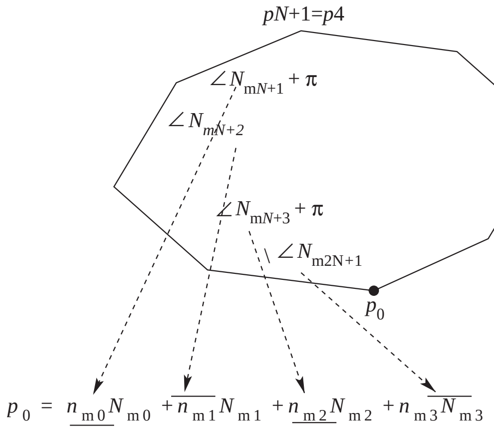

Fig. 3. Calculating coefficients of the vertex p_0. To shorten the notation each $N_i(e^{j\theta})$ is denoted by N_i.

of the vertex (see Fig. 3). Namely, denoting

$$\hat{a}_{k,i} = a_{(N+k+[(i-k) \bmod (N+1)]+1) \bmod (2N+2)}, \tag{7}$$

the vertices of the polygon are

$$p_k(e^{j\theta_0}) = \sum_{i=0}^{N} (\hat{a}_{k,i}\underline{n_{m_i}} + (1-\hat{a}_{k,i})\overline{n_{m_i}})N_{m_i}(e^{j\theta_0}),\quad k = 0,\dots,2N+1. \tag{8}$$

Note that when some of the coefficients of $f(e^{j\theta_0})$ are fixed, the number of vertices decreases to $2N'$ resulting in redundant p_k's in (8). Changing the notation and formulation to provide p_k's without redundancies is rather a straightforward task but will not be explicitly written down here in order to avoid complicated notation.

Note that the above procedure should be applied at each frequency where we want to obtain the polygon. However, for some widely used forms of $N_i(z)$ it is possible to avoid the computational burden of this procedure. For example, for $N_i(z) = z^{-i}$ and $N_i(z) = z^{-i}/\sum_{k=0}^{D} d_k z^{-k}$, following the Primary Algorithm in [5], the frequency axis can be divided into a finite number of intervals such that on each interval the vertices of the polygon, representing the uncertain polynomial in the numerator, are described by some fixed-coefficients polynomials and the order of these polynomials is fixed. The Primary Algorithm determines uniquely and analytically the above polynomials. Whenever possible, use this highly efficient algorithm in the solution.

4. Algorithm for model reduction

The solution of the problem (similar to several algorithms for model reduction of *fixed-coefficients* systems with infinity-norm cost function [12]) is based on linear semi-infinite programming (LSIP) technique. In order to use it, we should reformulate our problem as a linear programming problem with infinite number of linear constraints.

Consider a polygon representing some uncertain system, f.e. (1), at some frequency θ_0. Given the counterclockwise-ordered vertices of the polygon representing (1), say $p_i, i = 0,\dots,2N+1$, let us associate with each vertex p_i a set of unit-norm vectors

$$e^{j\phi_i},\quad \phi_i \in [\arg\{j(p_{i-1}-p_i)\},\ \arg\{j(p_i - p_{i+1})\}), \tag{9}$$

where $p_{-1} = p_{2N+1}$, $p_{2N+2} = p_0$, etc.

Let us also define the *signed distance* from a point, say x, to a polygon, say P, to be the distance from x to the closest point of P when x is outside P, and to be *minus the distance* from x to the closest point on the boundary of P when x is inside P.

Then, Theorem 1 from [7], slightly reformulated here, is:

Theorem 1 (*Dolgin and Zeheb [7]*). *Given an arbitrary point x and a polygon with vertices $p_l, l = 0,\dots,2N+1$ in the complex plane, calculate the inner products of the vector $(x - p_l)$ with all unit-norm vectors in the set $e^{j\phi_l}$, namely, $Re\{e^{-j\phi_l}(x-p_l)\}$, for all vertices p_l. The maximum of all these inner products is equal to the signed distance from x to the polygon.*

This theorem enables us to formulate the distance from a point to a polygon representing some uncertain system at some frequency as the maximum of an infinite number of functions $Re\{e^{-j\phi_l}(x-p_l)\}$. These functions are linear in the coefficients of the fixed-coefficients system (i.e., x) but nonlinear in the coefficients of the uncertain system (see (9)). This nonlinearity would cause a problem when formulating the distance from the vertices of $f(\mathrm{e}^{j\theta_0})$ to the polygon representing $\tilde{f}(\mathrm{e}^{j\theta_0})$.

However, using (5) and (8), which imply that the slope of the edge originating in p_l is $\arg(N_{m_l}(\mathrm{e}^{j\theta_0}))+a_l\pi$ when following the boundary of the polygon in the counterclockwise direction, we obtain

$$\arg\{j(p_l-p_{l+1})\}=\arg(N_{m_l}(\mathrm{e}^{j\theta_0}))+a_l\pi-\frac{\pi}{2} \tag{10}$$

and thus does not depend on the actual values of the coefficients n_i. Now, from (9), (10) and Theorem 1, we can formulate the signed distance from a point to a polygon as a maximum of *linear* functions of the coefficients of both uncertain systems:

$$\max Re\{\mathrm{e}^{-j\phi_l}(x-p_l)\},$$

$$\phi_l\in\Big[\arg(N_{m_{l-1}}(\mathrm{e}^{j\theta_0}))+a_{l-1}\pi-\frac{\pi}{2},\arg(N_{m_l}(\mathrm{e}^{j\theta_0}))+a_l\pi-\frac{\pi}{2}\Big),\quad l=\{1,\ldots,2(N+1)\}. \tag{11}$$

Proceeding further, for each $\theta\in[-\pi,\pi)$, obtain, using the procedure derived in Section 3, the sequences $a_i(\theta)\in\{0,1\}, m_i(\theta)\in\{0,\ldots,N\}, i=\{0,\ldots,2N+1\}$ corresponding to the uncertain system (1) and, similarly, the sequences $b_i(\theta)\in\{0,1\}, y_i(\theta)\in\{0,\ldots,X\}, i=\{0,\ldots,2X+1\}$ corresponding to the uncertain system (2). Then, due to (8), the vertices of the original system are

$$p_k(\mathrm{e}^{j\theta})=\sum_{i=0}^{N}(\hat{a}_{k,i}(\theta)\underline{n_{m_i(\theta)}}+(1-\hat{a}_{k,i}(\theta))\overline{n_{m_i(\theta)}})\times N_{m_i(\theta)}(\mathrm{e}^{j\theta}),\quad k=0,\ldots,2N+1 \tag{12}$$

and the vertices of the reduced system are

$$\tilde{p}_l(\mathrm{e}^{j\theta})=\sum_{i=0}^{X}(\hat{b}_{l,i}(\theta)\underline{x_{y_i(\theta)}}+(1-\hat{b}_{l,i}(\theta))\overline{x_{y_i(\theta)}})\times X_{y_i(\theta)}(\mathrm{e}^{j\theta}),\quad l=0,\ldots,2X+1. \tag{13}$$

Finally, the problem of finding the $\underline{x_i}$, $\overline{x_i}$ so as to minimize the distance between the original uncertain system (1) and the reduced uncertain system (2) as defined in Section 2, can be reformulated as a Linear Semi-Infinite Programming problem, by introducing the artificial quantity g:

minimize g, s.t.

$$Re\{\mathrm{e}^{-j\phi_k(\theta,s)}(\tilde{p}_l(\mathrm{e}^{j\theta})-p_k(\mathrm{e}^{j\theta}))\}\leqslant g, \tag{14a}$$

$$Re\{\mathrm{e}^{-j\tilde{\phi}_l(\theta,s)}(p_k(\mathrm{e}^{j\theta})-\tilde{p}_l(\mathrm{e}^{j\theta}))\}\leqslant g, \tag{14b}$$

$$\forall\theta\in[-\pi,\pi),\ s\in[0,1),\ k=1,\ldots,2N+2,\ l=1,\ldots,2X+2$$

$$\overline{x_i}-\underline{x_i}\geqslant 0,\quad i=0,\ldots,X. \tag{14c}$$

Note that (see (7), (11), (12), (13)):

$$\phi_k(\theta,s)=\Big(\arg(N_{m_{k-1}(\theta)}(\mathrm{e}^{j\theta}))+a_{k-1}(\theta)\pi-\frac{\pi}{2}\Big)(1-s)+\Big(\arg(N_{m_k(\theta)}(\mathrm{e}^{j\theta}))+a_k(\theta)\pi-\frac{\pi}{2}\Big)s, \tag{15a}$$

$$\tilde{\phi}_l(\theta,s)=\Big(\arg(X_{y_{l-1}(\theta)}(\mathrm{e}^{j\theta}))+b_{l-1}(\theta)\pi-\frac{\pi}{2}\Big)(1-s)+\Big(\arg(X_{y_l(\theta)}(\mathrm{e}^{j\theta}))+b_l(\theta)\pi-\frac{\pi}{2}\Big)s, \tag{15b}$$

$$p_k(\mathrm{e}^{j\theta})=\sum_{i=0}^{N}(\hat{a}_{k,i}(\theta)\underline{n_{m_i(\theta)}}+(1-\hat{a}_{k,i}(\theta))\overline{n_{m_i(\theta)}})N_{m_i(\theta)}(\mathrm{e}^{j\theta}), \tag{15c}$$

$$\tilde{p}_l(\mathrm{e}^{j\theta})=\sum_{i=0}^{X}(\hat{b}_{l,i}(\theta)\underline{x_{y_i(\theta)}}+(1-\hat{b}_{l,i}(\theta))\overline{x_{y_i(\theta)}})X_{y_i(\theta)}(\mathrm{e}^{j\theta}), \tag{15d}$$

$$\hat{a}_{k,i}(\theta)=a_{(N+k+[(i-k)\bmod(N+1)]+1)\bmod(2N+2)}(\theta), \tag{15e}$$

$$\hat{b}_{l,i}(\theta)=b_{(X+l+[(i-l)\bmod(X+1)]+1)\bmod(2X+2)}(\theta). \tag{15f}$$

Note, that the constraints (14a) demand the polygon of the reduced system to be maximally inside the polygon of the original system at each frequency, while the constraints (14b) demand, vice a versa, the polygon of the original system to be maximally inside the polygon of the reduced system at each frequency. Moreover, if the value set of the reduced system is desired to be contained in the value set of the original system at all these frequencies, then the g in the constraints (14a) should be replaced by 0. If, on the contrary, the value set of the original system is desired to be contained in the value set of the reduced system, then the g in the constraints (14b) should be replaced by 0. Note also, that when some of the coefficients of (1) or (2) are fixed then some of the above constraints are redundant and thus can be removed.

This problem is a linear semi-infinite programming problem (i.e., linear programming problem with infinite number of linear constraints) which can be solved by well-known efficient techniques [1,8,11,13,19]. Note, that most of the techniques are extensions of the algorithms for regular linear programming case and have similar properties. For example, there exists simplex extension (see [1]) for *semi-infinite* linear programming which is rather efficient and provides the optimal solution. Also, a nonlinear system of equations based on Kuhn–Tucker conditions may be derived, from which an optimal solution of the LSIP problem may be constructed (see [11]). Note, that the solution of general (non-linear) semi-infinite programming problems is also based on Kuhn–Tucker conditions, but, the resulting system is much more complicated and the optimality of the resulting solution cannot be guaranteed in that (nonlinear) case. Additionally, there exist interior-point methods for linear semi-infinite programming problem (see [8]).

5. Example

In this example we consider an order reduction of a linear-in-parameters uncertain system which is IIR as well as polytopic.

Let the original transfer function be given by

$$f_0(z^{-1}) = \frac{z^{-18}}{z^{-3}+2z^{-2}+5} + 3z^{-17} - 4z^{-16} + 15z^{-15} + 13z^{-14} + 10(z^{-13}+z^{-8}) - 8\,\frac{z^{-12}}{z^{-2}+z^{-1}+3} + 23z^{-11} + 12z^{-10} + 1z^{-9} + 18z^{-8} + 25z^{-7} - 13z^{-6} + 34z^{-5} + 42z^{-4} - 32z^{-3} + 160z^{-2} + 200z^{-1} + 100.$$

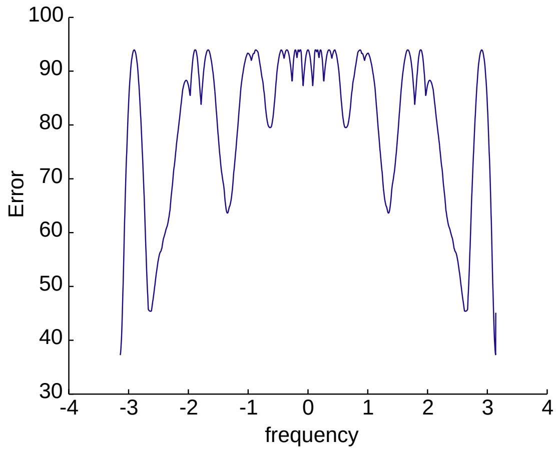

Fig. 4. Approximation of linear-in-parameters uncertain system. Reduction from order 18 to order 5.

We introduce 20% uncertainty in all the coefficients (not including the fixed denominators) and we seek for the reduced linear-in-parameters uncertain system of order 5 of the form

$$\sum_{i=0, i\neq 4}^{5} \tilde{a}_i z^{-i} + \tilde{a}_4 \frac{z^{-4}}{z^{-1}+3} \qquad (16)$$

approximating the complex valued frequency behaviour of the original transfer function.

Applying the algorithm of model reduction derived in this paper we obtain

$$\tilde{f}(z^{-1}) = \sum_{i=0, i\neq 4}^{5} \tilde{a}_i z^{-i} + \tilde{a}_4 \frac{z^{-4}}{z^{-1}+3}$$

where

$$\tilde{a}_0 = [64.69, 121.45], \quad \tilde{a}_1 = [164.14, 247.56],$$

$$\tilde{a}_2 = [108.98, 169.10], \quad \tilde{a}_3 = [-29.26, -1.37],$$

$$\tilde{a}_4 = [127.13, 127.13], \quad \tilde{a}_5 = [37.89, 111.36].$$

The error of approximation is shown in Fig. 4. In Fig. 5 we show both the original (solid polygons) and the reduced (dashed polygons) polynomials on some grid of frequencies in $[0, \pi]$ range. The complementary interval $[-\pi, 0]$ may be formed by symmetry considerations. Qualitatively, we can conclude that the

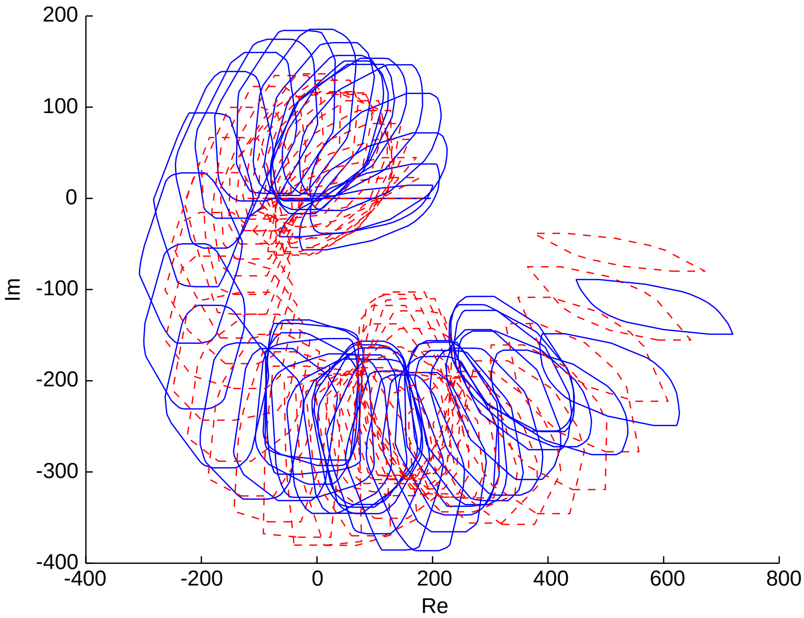

Fig. 5. Approximation of linear-in-parameters uncertain system. Reduction from order 18 to order 5.

reduced order polygons closely track the pattern of the original polygons.

6. Conclusion

In this paper we discuss and define the meaning of model reduction for uncertain systems. We propose a new method for approximation of a given uncertain linear-in-parameters system by uncertain linear-in-parameters system of a pre-determined lower order. Our method may be considered as a generalization of the Complex Chebyshev Approximation problem which makes the latter applicable to uncertain systems. A general algorithm for obtaining the value set of uncertain linear-in-parameters system is proposed. In special cases of systems with specific frequently used transfer functions, the algorithm becomes even more effective using some recently derived results. A major advantage of the method derived in the present paper is its formulation as a semi-infinite linear programming problem. This significantly reduces the computational complexity. Finally, an illustrative example indicates that our method is capable of providing effective approximation to the original uncertain polynomial by a lower order uncertain polynomial, with approximately similar uncertainty structure.

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