Inference Rules for Temporal Next-Time Logic

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The Next-Time Temporal Logic LTN_{ext} is originated from AI and CS. The formulas of LTN_{ext} are merely all propositional formulas with Boolean operations and operations \Box — always in future and \bigcirc — tomorrow, next time. The logic LTN_{ext} is simply the set of all temporal formulas valid on the linearly ordered set of all natural numbers. We study properties of the Next-Time Temporal Logic LTN_{ext} focusing on the question which inference rules are correct temporal clauses. Firstly we distinguish with examples admissible and derivable temporal rules. Derivable rules can be recognized by standard decision techniques for this logic which is not a case for admissible rules. In order to describe admissible in LTN_{ext} rules we construct so-called k-exhaustive temporal models $M(k, LTN_{ext})$ and prove

THEOREM 1. Any given rule r is admissible for the temporal logic LTN_{ext} iff, for any k, r is valid in the frame of the k-exhaustive model $M(k, LTN_{ext})$ for any definable valuation V in $M(k, LTN_{ext})$.

This description is a good semantic criterion which however does not allow us to recognize effectively all admissible rules because the description uses infinite models and the notion of definable valuations. For to evolve our technique till reaching an algorithmic criterion we need so called normal reduced forms for rules of temporal logic and a reduction of arbitrary rules to normal forms.

THEOREM 2. There exists an algorithm which for any given temporal inference rule r constructs its normal reduced form nrf(r).

And applying this result and our previous semantic description for admissible rules of temporal logic LTN_{ext} we drive

THEOREM 3. The temporal logic LTN_{ext} is decidable w.r.t. admissible rules, i.e. there is an algorithm which for any given inference rule r determines whether r is admissible in LTN_{ext} .

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