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Interfaces in Solutions of Diffusion–Absorption Equations in Several Space Dimensions

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We study the properties of interfaces in solutions of the Cauchy problem for the nonlinear degenerate parabolic equation

$$u_t = \Delta\, u^m - a\, u^p \qquad \text{in } \mathbf{R}^n \times (0, T], \quad n \geq 1$$

with the parameters $m > 1$, $p > 0$, $a > 0$ satisfying the condition $m + p \geq 2$. We show that the velocity of the interface $\Gamma(t) = \partial\overline{\{\operatorname{supp} u(x,t)\}}$ is given by the formula

$$\mathbf{v} = \left[-\frac{m}{m-1} \nabla\, u^{m-1} + \nabla\, \Pi \right] \Big|_{\Gamma(t)}$$

where Π is the solution of the degenerate elliptic equation

$$\operatorname{div}(u \nabla\, \Pi) = a\, u^p, \qquad \Pi = 0 \text{ on } \Gamma(t).$$

The first term expresses the classical Darcy law, while the second one models the presence of an "external force" that makes the interface to move in the inward direction. We give explicit formulas which represent the interface $\Gamma(t)$ as a bijection from $\Gamma(0)$. It is proved that the solution u and its interface $\Gamma(t)$ are analytic functions of time t and that they preserve the initial regularity in the spatial variables. We also show that the regularity of the interface velocity $\mathbf{v}$ with respect to the spatial variables is better than it was at the initial instant.

The analysis is based on a special coordinate transformation (a local version of Lagrangian coordinates) that renders the free boundary stationary. The rigorous proofs for the cases $n = 1, 2, 3$ are given in [**2**]. In the case $n = 1$ the interface equation coincides with that derived in [**1**] by means of the method of intersection comparison.

References

[1] V. A. Galaktionov, S. I. Shmarev, J. L. Vazquez, *Regularity of interfaces in diffusion processes under the influence of strong absorption*, Arch. Ration. Mech. Anal. **149** (1999), no. 3, 183–212.

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[2] S. I. Shmarev, *Interfaces in multidimensional diffusion equations with absorption terms*, Nonlinear Anal. **53** (2003) 791–828.

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