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C^0-Topology in Morse Theory

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Let f be a Morse function on a closed manifold M, and v be a Riemannian gradient of f satisfying the transversality condition. The classical construction (due to Morse, Smale, Thom, Witten), based on the counting of flow lines joining critical points of the function f associates to these data the Morse complex.

We introduce a new class of vector fields (*f-gradients*) associated to a Morse function f. This class is wider than the class of Riemannian gradients and provides a natural framework for the study of the Morse complex. Our construction of the Morse complex does not use the counting of the flow lines, but rather the fundamental classes of the stable manifolds, and this allows to replace the transversality condition required in the classical setting by a weaker condition on the f-gradient (*almost transversality condition*) which is C^0-stable. We prove then that the Morse complex is stable with respect to C^0-small perturbations of the f-gradient, and study the functorial properties of the Morse complex.

We discuss the generalizations of these results in the framework of Morse–Novikov theory of circle-valued Morse maps.